ONEforest

WP8: Multi-Criteria Decision Support System

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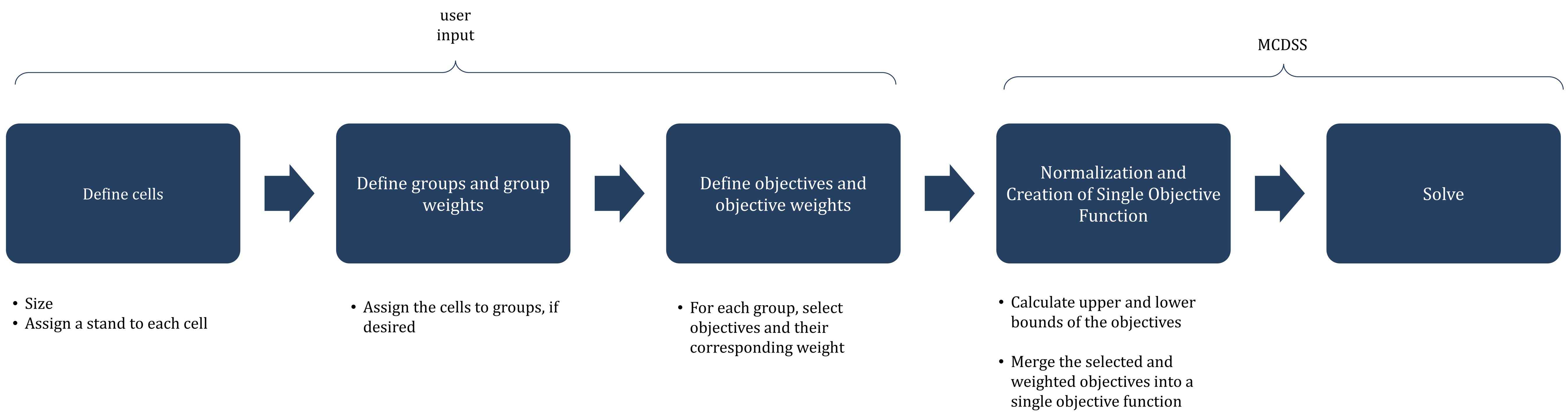
Solution Algorithm



Mathematical Model

Maximize weighted sum objective:

$$\max y^*_{o,g} = \frac{y_{o,g} - \underline{Y_{o,g}}}{\overline{Y_{o,g}} - \underline{Y_{o,g}}} \qquad \forall\, g \in G, o \in O_g$$

$$y_{(i,free),g} = v_{i,g} \qquad \forall\, g \in G, i \in I_g^{free}$$

Indicator constraints:

harvested timber
$$v_{(harvested\ timber),g} = \sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{HarvestedTimber}_{c,m,t} \cdot F_c \qquad \forall\, g \in G$$

periodic annual increment
$$v_{(periodic\ annual\ increment),g} = \sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{PeriodicAnnualIncrement}_{c,m,t} \qquad \forall\, g \in G$$

Carbon-sequestration
$$v_{(c-sequestration),g} = \sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{Carbon}_{c,m,t} \cdot F_c \qquad \forall\, g \in G$$

Standing timber
$$v_{(standing\ timber),g} = \sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{Standing}_{c,m,t} \cdot F_c \qquad \forall\, g \in G$$

Large trees
$$v_{(large\ trees),g} = \sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{Trees}_{c,m,t} \cdot F_c \qquad \forall\, g \in G$$

... ...

Shannon Index
$$v_{(shannon\ index),g} = -\sum_{t\in T}\sum_{c\in C_g}\sum_{m\in M}\sum_{s\in S} x_{c,m} \cdot F_{c,m,s,t} \cdot \ln\left(x_{c,m} \cdot F_{c,m,s,t}\right) \qquad \forall\, g \in G$$

=> Approximation through linearization necessary!

Assumption: shannon index reaches the maximum if following statement applies:

$$\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot F_{c,m,s,t} = \frac{1}{|S|} \qquad \forall t \in T, s \in S$$

⬇

$$\min \hat{y} = \sum_{t\in T}\sum_{s\in S}(u_{t,s} + o_{t,s})$$

s.t.:

$$\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot F_{c,m,s,t} + u_{t,s} - o_{t,s} = \frac{1}{|S|} \qquad \forall s \in S, t \in T$$

$$u_{t,s} \le s_{t,s} \qquad \forall s \in S, t \in T$$
$$o_{t,s} \le 1 - s_{t,s} \qquad \forall s \in S, t \in T$$
$$0 \le o_{t,s}, u_{t,s} \le 1 \qquad \forall s \in S, t \in T$$
$$s_{t,s} \in \{0;1\} \qquad \forall s \in S, t \in T$$

Selecting one management option for each cell

$$\sum_{m\in M} x_{c,m} = 1 \qquad \forall c \in C$$

$$x_{c,m} \in \{0;1\} \qquad \forall c \in C, m \in M$$

Hard constraints, e.g.

$$\sum_{c\in C_g}\sum_{m\in M} x_{c,m} \cdot B^{Trees}_{c,m,t} \overset{\le}{\ge} Limit \qquad \forall\, t \in T$$

...

Indices				
t - index of the period	c – index of the cell	g – index of a group	m – index of management option	s – index of species

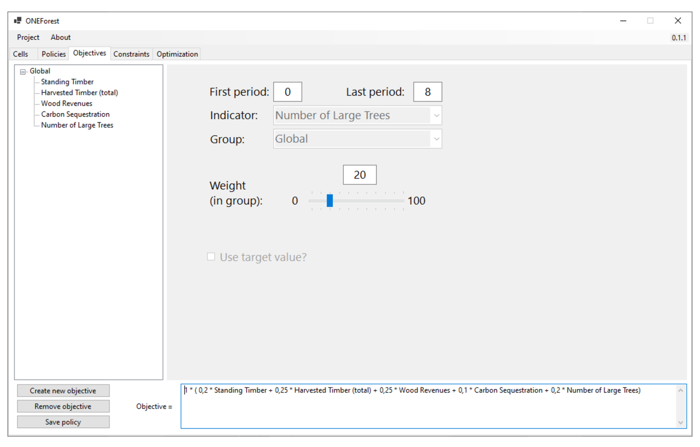
Sets					
M	Management options	C	Cells	O_g	objectives o considered in group g
I_g^{free}	indicators i considered as objective in group g	C_g	Cells in group g		
T	Periods	G	Groups		

Input: Parameters	
$\underline{Y_{o,g}}$	lower bound for objective o in group g
$\overline{Y_{o,g}}$	upper bound for objective o in group g
$B^{HarvestedTimber}_{c,m,t,a}$	Harvested amount of timber in m^3 of species s of assortment a in period t if management option m is applied in cell c
$B^{PeriodicAnnualIncrement}_{c,m,t}$	Volume increment per period t for cell c with management option m
$B^{Carbon}_{c,m,t}$	Carbon sequestration in period t if management option m is applied in cell c
$B^{standing}_{c,m,t}$	Standing timber in period t if management option m is applied in cell c
$B^{Trees}_{c,m,t}$	Number of large trees in period t if management option m is applied in cell c
F_c	Size of cell c in hectare

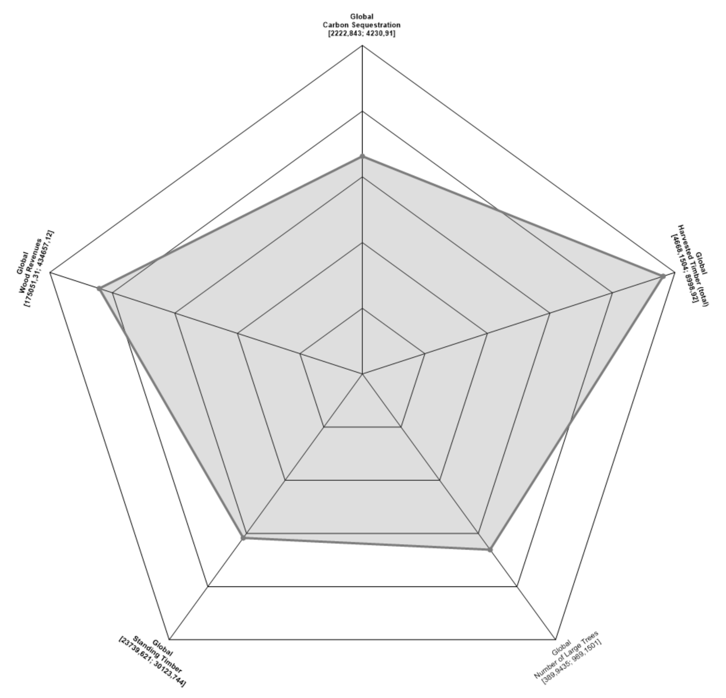
Output: Variables	
$y^*_{o,g}$	normalized value of objective o in group g
$v_{i,g}$	value of indicator i in group g
$x_{c,m}$	=1, if management option m is applied for cell c, otherwise 0

Constraints / Target value

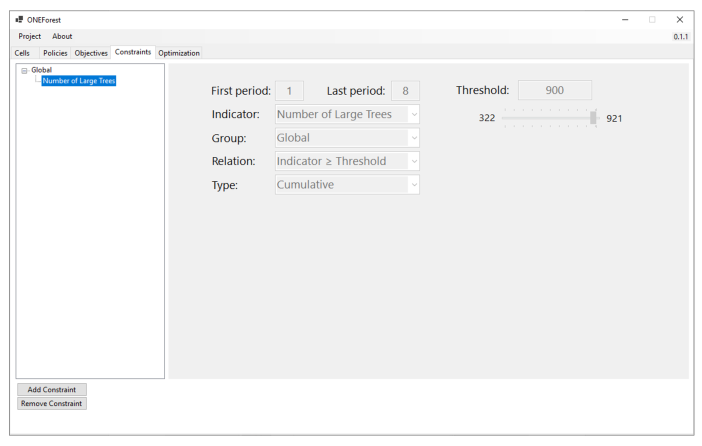
Free optimization



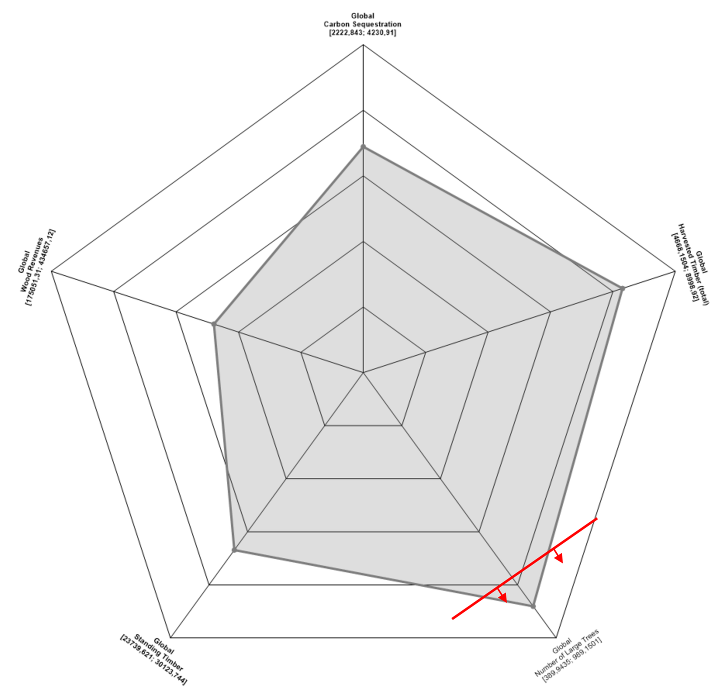
Free optimization maximizes the weighted, normalized objective function without specifying user constraints.



With user constraints



User constraints allow the user to set hard limits for indicators, which can be derived from legal requirements, for example. If a value of at least 900 large trees is specified (red line in figure), for example, the number of large trees increases compared to free optimization without constraints. This is at the expense of wood revenues and harvested timber. Carbon sequestration and standing timber increase.



Target value:

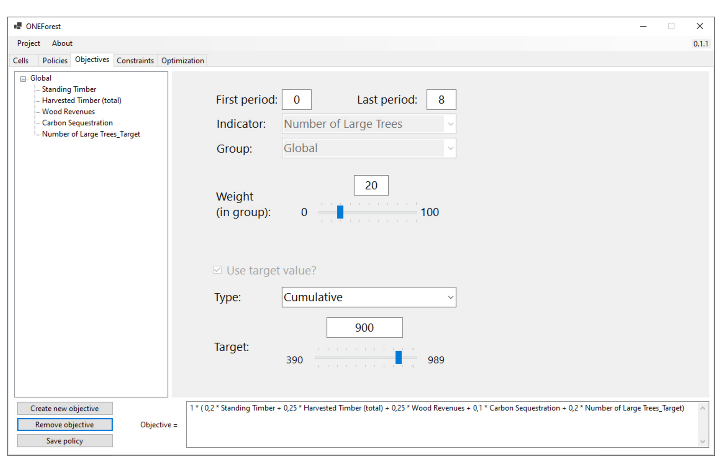
Output: Variables	
d^-	Deviation in case of undercutting the target value
d^+	Deviation in case of exceeding the target value
v	auxiliary variable to make sure that d^+ and d^- are not both greater than 0

$$IndicatorValue + d^- - d^+ = TargetValue$$

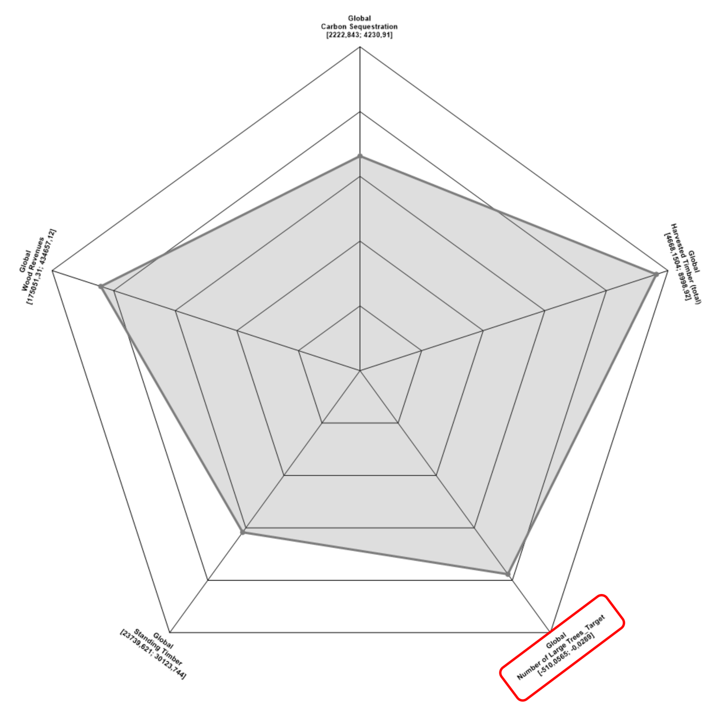
$$d^- \le MaxIndicatorValue \cdot v$$
$$d^+ \le MaxIndicatorValue \cdot (1 - v)$$
$$v \in \{0;1\}, d^- \ge 0, d^+ \ge 0$$

Cumulative targets
(e.g., 900 large trees over the whole planning horizon)

Periodically targets
(e.g., 100 large trees in each period of time)



Target values can be understood as soft constraints. The deviation from a target value is minimized. However, the target value is not necessarily achieved. If a target value of 900 trees is set, the weighted sum of the target functions is again maximized, but the number of trees is not increased at the expense of the other indicators.



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